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Autocorrelation and cross-correlation in time series of homicide and attempted homicide

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HIGHLIGHTS

- We establish the relationship between homicides and attempted homicides by DFA, DCCA, and DCCA cross-correlation coefficient.
- DCCA cross-correlation coefficient identifies a positive cross-correlation.
- The DFA analysis can be more informative depending on time scale (short or long).
- For short scale DFA did not identify auto-correlations, and for long scales DFA presents a persistent behavior.

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ABSTRACT

We propose in this paper to establish the relationship between homicides and attempted homicides by a non-stationary time-series analysis. This analysis will be carried out by Detrended Fluctuation Analysis (DFA), Detrended Cross-Correlation Analysis (DCCA), and DCCA cross-correlation coefficient, $\rho_{DCCA}(n)$. Through this analysis we can identify a positive cross-correlation between homicides and attempted homicides. At the same time, looked at from the point of view of autocorrelation (DFA), this analysis can be more informative depending on time scale. For short scale (days), we cannot identify auto-correlations, on the scale of weeks DFA presents anti-persistent behavior, and for long time scales ($n > 90$ days) DFA presents a persistent behavior. Finally, the application of this new type of statistical analysis proved to be efficient and, in this sense, this paper can contribute to a more accurate descriptive statistics of crime.

1. Introduction

Due to political, economic and social factors, crime has been studied and statistically modeled by many researchers. For example, it is possible to statistically measure the connection between unemployment and crime [1–3], the correlation between firearms and homicides [4], make a descriptive study of homicides considering author and victim [5], evaluate crime rates through probabilistic models [6], perform a temporal and spatial study of crime [7,8], analyze the flux of tourists and increase in crime [9], simulate computationally criminal activity in an urban environment [10], among others. In this way it is possible to say that crime can be modeled based on the author–victim profile, time, and geographic location, as well as, other variables. This paper aims to detect and measure the auto-correlation and the cross-correlation of homicides and attempted homicides in the city of Salvador, located in the state of Bahia (Brazil). Salvador (12°59′S, 38°29′W) is one of the largest cities in Brazil, with more than 2.7 million people, and with 3787 people per square kilometer [11]. It is worth mentioning that Salvador will host six matches of the 2014 FIFA World Cup Brazil.

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The crime was studied in terms of homicides and attempted homicides because these are crimes against people and are widely used in empirical studies about the determinants of crime. In this sense Fig. 1 shows the time-series of homicides and attempted homicides per 100,000 citizens. In this figure we can see large irregularities (unpredictable), characteristic of a nonlinear system. Such systems have been studied from the point of view of complex systems. The complex systems are studied in many areas of the natural sciences, mathematics, and the social sciences [12–14]. Complex systems have nonlinear behavior, and can be studied by taking into account the properties of fractals [15], such as self-affinity in time series. If, for example, in a given time-series $\{u(i)\}$ [16] self-affinity appears, then long range power-law correlations are present [17–19]. This makes the study of complex systems very interesting, because it is possible to identify a universality in different kinds of problems [20,21]. It is known that, in the real world, data are highly non-stationary [22], and many conventional methods of analysis are not suited for non-stationary time-series [23].

For non-stationary time-series, we did our analysis in the point of view of Detrended Fluctuation Analysis, DFA [24], Detrended Cross-Correlation Analysis, DCCA [25], and DCCA cross-correlation coefficient, ρ_{DCCA} [26]. Thus, the rest of the paper is laid out as follows: Section 2 provides a brief theoretical review of these methods. Section 3 describes the data used in this paper and presents our results and, finally, Section 4 concludes the paper.

2. Brief review of DFA, DCCA, and ρ_{DCCA}

There are situations where a given observable $u(i)$ is measured at successive time intervals, forming a time-series $\{u(i)\}$ [16]. Some strategies for time-series analysis have been developed [22,23,27–38]. Today, one of the most popular methods for nonstationary time-series analysis is the Detrended Fluctuation Analysis (DFA) and will be briefly presented below.

2.1. The DFA method [24]

The DFA method was developed to analyze long-range power-law correlations in non-stationary systems like in Refs. [24, 29,33,39–47], among others. The DFA method involves the following steps: (see Fig. 2) or Ref. [48].

1. Consider a correlated signal $u(i)$ (daily homicides, attempted homicides), where $i = 1, \dots, N_{\max}$ (the total number of points in the series). We integrate the signal $u(i)$ and obtain $y(k) = \sum_{i=1}^{k} u(i) - \langle u \rangle$, where $\langle u \rangle$ stands for the average of u;
2. The integrated signal $y(k)$ is divided into boxes of equal length n;
3. For each n-size box, we fit $y(k)$, using a polynomial function of order l, which represents the trend in the box. The y coordinate of the fitting line in each box is denoted by $y_n(k)$, since we use a polynomial fitting of order l, we denote the algorithm by DFA-l;
4. The integrated signal $y(k)$ is detrended by subtracting the local trend $y_n(k)$ in each box (of length n);
5. For a given n-size box, the root-mean-square fluctuation, $F(n)$, for this integrated and detrended signal is given by

$$F_{\text{DFA}}(n) = \sqrt{\frac{1}{N_{\max}} \sum_{k=1}^{N_{\max}} [y(k) - y_n(k)]^2}. \tag{1}$$

6. The above computation is repeated for a broad range of scales (n-sizes box) to provide a relationship between $F(n)$ and the box size n.

In accordance with Refs. [24,48], in this paper we used a polynomial fitting of order 1, with $n = 4$ for the smallest and $n = N_{\max}/4$ for the largest box width. Thus, the DFA method provides a relationship between $F_{\text{DFA}}(n)$ (root mean square fluctuation) and the time scale n, characterized by a power-law:

$$F_{\text{DFA}}(n) \propto n^{\alpha}. \tag{2}$$

In this way, α is the scaling exponent, a self-affinity parameter representing the long-range power-law correlation properties of the signal; such that if $\alpha = 0.5$, then the signal is uncorrelated; if $\alpha < 0.5$, then the correlation in the signal is anti-persistent; and if $\alpha > 0.5$, then the correlation in the signal is persistent.

However, we know that many observables can be measured and recorded simultaneously, at successive time intervals, forming time-series with the same length N [16]. For example, if we have two time-series, then the analysis of the cross-correlation between these time-series can be carried out. Naturally, in the next section, we apply a generalization of the DFA method, called detrended cross-correlation analysis (DCCA), to study the long range cross-correlations in the presence of non-stationarity [49–59].

2.2. The DCCA method [25]

Given two time-series, $\{u_1(i)\}$ and $\{u_2(i)\}$, we compute the integrated signals $R_1(k) \equiv \sum_{i=1}^{k} u_1(i)$ and $R_2(k) \equiv \sum_{i=1}^{k} u_2(i)$, where $k = 1, \dots, N_{\max}$. Next, we divide the entire time-series into $(N-n)$ overlapping (or not) boxes, each containing $(n+1)$ values. For both time series, in each box that starts at i and ends at $i+n$, we define the local trend, $\widetilde{R}_{1,i}(k)$ and $\widetilde{R}_{2,i}(k)$ ($i \le k \le i+n$), to be the ordinate of a linear least-squares fit. We define the detrended walk as the difference between the original walk and the local trend. Next, we calculate the covariance of the residuals in each box $f^2_{\text{DCCA}}(n, i) \equiv 1/(n+1) \sum_{k=i}^{i+n} (R_1(k) - \widetilde{R}_{1,i}(k))$

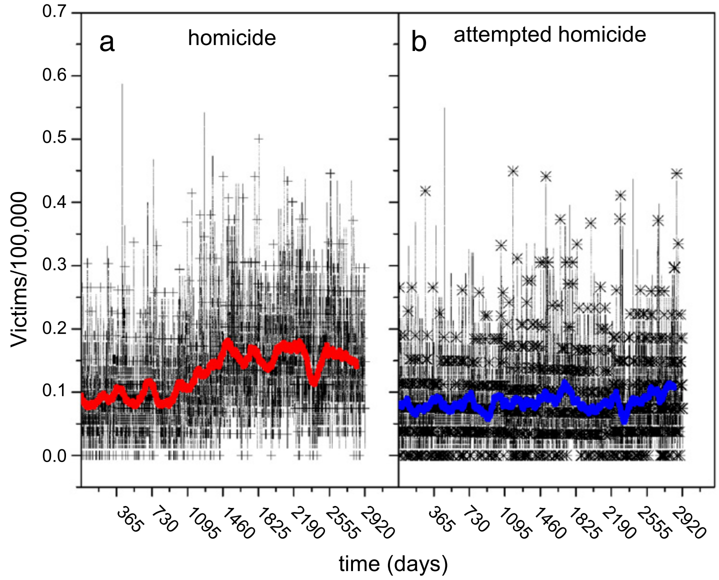

Fig. 1. (Color online): time series of victims/100,000 citizens in Salvador between January 2004 and December 2011 for: (a) homicide and (b) attempted homicide. Continuous line represents the moving average for $n = 90$ days.

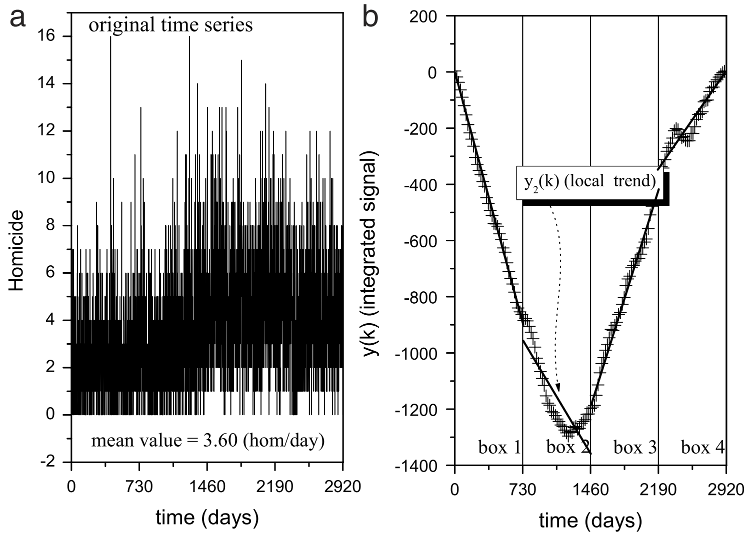

Fig. 2. (a) Time series of daily homicides in the city of Salvador, Bahia (Brazil) for data recorded between January 2004 and December 2011. (b) Integrated time series $y(k)$ of the original data, showing the application of the DFA algorithm. In this case the continuous line in each box, with $n = N_{max}/4 = 730$, represents the linear adjust (detrended).

$(R_2(k) - \widetilde{R}_{2,i}(k))$. Finally, the detrended covariance function is calculated by summing over all overlapping $(N - n)$ boxes of size n,

$$F^2_{DCCA}(n) \equiv (N - n)^{-1} \sum_{i=1}^{N-n} f^2_{DCCA}(n, i). \tag{3}$$

If self-affinity appears, then a power-law exists in the cross-correlations, in other words,

$$F^2_{DCCA}(n) \sim n^{2\lambda}, \tag{4}$$

where λ is the long range power-law cross-correlation exponent. Supposing $(R_1(k) = R_2(k))$, the detrended covariance $F^2_{DCCA}(n)$ reduces to the detrended variance $F^2_{DFA}(n)$ used in the DFA method. According to Podobnik and Stanley [25], in general, λ tends to be the mean value of DFA exponents, e.g.,

$$\lambda \approx \frac{(\alpha_1 + \alpha_2)}{2}. \tag{5}$$

Table 1
ρ_{DCCA} in terms of the level of cross-correlation.

ρ_{DCCA}	Condition
1	Perfect cross-correlation
0	No cross-correlation
−1	Perfect anti cross-correlation

Table 2
Descriptive statistics for homicides and attempted homicides registered in the city of Salvador, the state of Bahia (Brazil), from January 2004 to December 2011.

Measure	Homicide	Attempted homicide
Average (sd)	3.60 (2.52)	2.34 (2.05)
Sum	10,529	6853
Skewness	0.90	1.37
Kurtosis	0.99	2.71
Minimum (maximum)	0 (16)	0 (15)

We can point out other methods of estimation of the λ [60–63]. The exponent λ quantifies the long range power-law correlations and also identifies seasonality [59]. But, λ does not quantify directly the level of cross-correlation. In this case it is possible to quantify the level of cross-correlation with the DCCA cross-correlation coefficient, defined as the ratio between the detrended covariance function $F^2_{DCCA}(n)$ and the detrended variance function $F_{DFA}(n)$, which will be presented below.

2.3. The DCCA cross-correlation coefficient [26]

The cross-correlation coefficient was defined in order to quantify the level of cross-correlation between non-stationary time-series. The DCCA cross-correlation coefficient is defined as the ratio between the detrended covariance function F^2_{DCCA} and the detrended variance function F_{DFA} of $\{u_1(i)\}$ and $\{u_2(i)\}$, i.e.,

$$\rho_{DCCA}(n) \equiv \frac{F^2_{DCCA}(n)}{F_{DFA_{\{u_1\}}}(n)\, F_{DFA_{\{u_2\}}}(n)}. \tag{6}$$

Eq. (6) leads us to a new scale of cross-correlation in non-stationary time-series. Here, $\rho_{DCCA}(n)$ is a dimensionless coefficient that ranges between $-1 \leq \rho_{DCCA}(n) \leq 1$. A value of $\rho_{DCCA}(n) = 0$ means there is no cross-correlation, and it splits the level of cross-correlation between the positive and the negative case (see Table 1).

The detrended coefficient $\rho_{DCCA}(n)$ has been tested on simulated and real time-series [26,64–69]. Besides, the statistical results of $\rho_{DCCA}(n)$ have been compared with the Pearson correlation coefficient for time-series in the US stock market [70]. Also, it is possible to think in terms of the derivative of $\rho_{DCCA}(n)$ coefficient, with a well-defined relationship between α_{DFA} and λ_{DCCA} [71].

3. Data and results

These data are based on the police records made daily in police stations by citizens and obtained via Secretariat of Public Security of the State of Bahia (Brazil), from January 2004 to December 2011 [72]. As a first form of data analysis, Fig. 1 presents the daily rate of victims. There is an evident growth of the homicide rate (+), but we cannot say the same thing for attempted homicides ($*$). Looking at Fig. 1, we do not know whether to say there is a relationship between these variables. In order to determine whether there is such a relationship, we present the descriptive statistics in Table 2. The time-series for homicides (attempted homicides) has an average of 3.60 (2.34) occurrences per day. Both variables showed positive skewness, and the highest occurrence of homicides (attempted homicides) is 16 (15) victims per day. As per information about the behavior of these non-stationary time-series, the continuous line in Fig. 1 represents the moving average (with $n = 90$) of the rate of victims. This analysis smooths the stronger oscillations and makes it easier to understand the behavior, positive or negative trends, of the variables. Fig. 3, shows a classical signal analysis with the autocorrelation function (periodogram) and the Fourier transform (amplitude). We can identify a clear periodicity, seven days, but for long-time scale it was not possible to identify other periodicities. Our proposal is to analyze these time-series with new feature, and if we analyze these time series by DFA, DCCA, and ρ_{DCCA}, then we can make new conclusions, as will be shown here.

Fig. 4 and Table 3 show the values of the DFA exponent for homicides, α_+, attempted homicides, α_*, and DCCA cross-correlation exponent, λ. In this way we can estimate if the time-series exhibit persistent, anti-persistent, or uncorrelated behavior.

For both time-series (crimes) we can identify interesting situations. Firstly, there is some cross-correlation between the series, because $F_{DCCA}(n) \neq 0$ ($\circ$, in Fig. 4). Next, we can identify seasonal components (vertical lines), i.e., for: $n \leq 7$, $7 \leq n \leq 30$, $30 \leq n \leq 90$, $90 \leq n \leq 365$, and finally $n \geq 365$ days. Even more, there is a transition of behavior between these series, depending on the time scale. Therefore, the DFA auto-correlation exponent can be $\alpha_{DFA} \approx 0.5$ (no memory), $\alpha_{DFA} < 0.5$ (anti-persistent), or $\alpha_{DFA} > 0.5$ (persistent). In other words, for $n \leq 7$, the crime occurs randomly, i.e., $\alpha_{DFA} \simeq 0.5$ (see

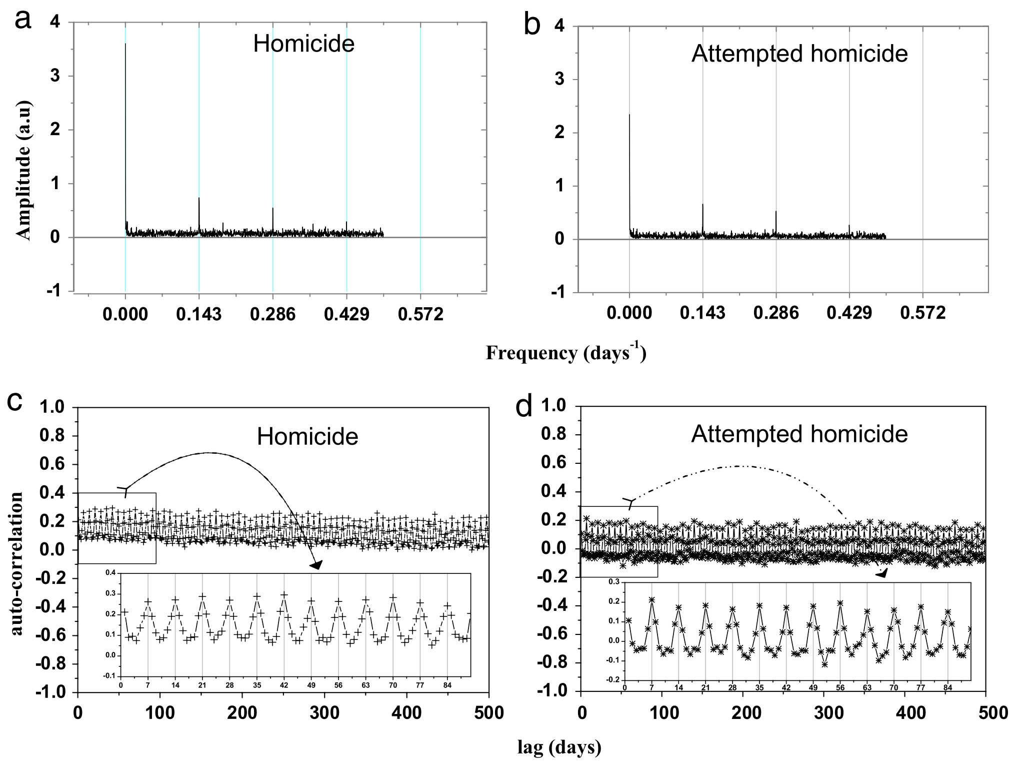

Fig. 3. Traditional signal process for homicides (left) and attempted homicides (right). In this case (a) and (b) represent the amplitude spectrum for Fourier transform and (c) and (d) the auto-correlation function. Vertical lines in this figure represent 7 days.

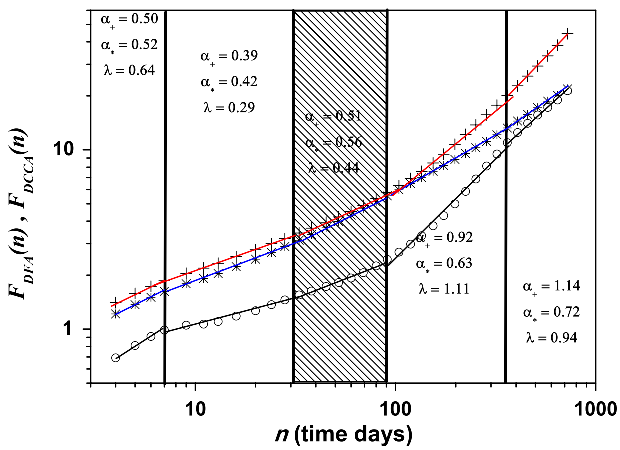

Fig. 4. Detrended variance $F_{\text{DFA}}(n)$ for homicide (+), attempted homicide (∗), and detrended covariance $F_{\text{DCCA}}(n)$ between homicides and attempted homicides (○), as functions of n. Vertical lines represent the time scale. At each interval we print the values of the coefficients.

Table 3). For $7 \leq n \leq 30$, the crime is anti-persistent, this means that homicides had $\alpha_+ = 0.39$ and $\alpha_* = 0.42$ for attempted homicides. Now, if we look at the crosshatched column in Fig. 4, between $30 \leq n \leq 90$ days, there is an anti-persistent/persistent transition, and $n = 90$ apparently represents the point for this transition (see Fig. 4).

It is worth mentioning here that the effect of trends on DFA was studied in Ref. [73]. In this paper Hu et al. showed that the DFA method performs better than the standard R/S analysis to quantify the scaling behavior of noisy signals for a wide range of correlations, and we estimate the range of scales where the performance of the DFA method is optimal.

Taking into account that we have identified a cross-correlation between the time-series of homicides and attempted homicides, by F_{DCCA}, now we apply DCCA cross-correlation coefficient, $\rho_{\text{DCCA}}(n)$, in order to quantify the level of cross-correlation between these crimes (new methodology). The statistical analysis of the relationship between homicides and attempted homicides has identified persistent and positive cross-correlations at different time scales, Fig. 5. Specifically the results show that the ρ_{DCCA} oscillates around 0.29, with $\rho_{\text{DCCA}}(7) = 0.32$, $\rho_{\text{DCCA}}(90) = 0.18$ (minimum value), and $\rho_{\text{DCCA}}(> 365) = 0.47$ (maximum value).

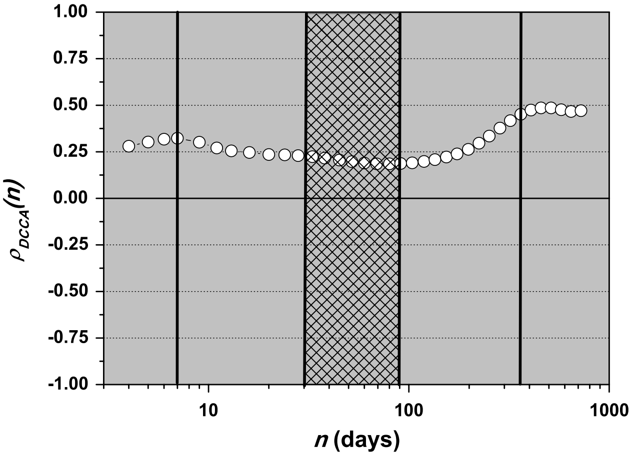

Fig. 5. DCCA cross-correlation coefficient $\rho_{\mathrm{DCCA}} \times n$ for cross-correlation between homicides and attempted homicides in Salvador.

Table 3
Auto-correlation exponent α_{DFA}, for homicides (+) and attempted homicides (∗), and cross-correlation exponent λ_{DCCA}.

Time scale	α_+	α_*	λ
$n \leq 7$	0.50	0.52	0.64
$7 \leq n \leq 30$	0.39	0.42	0.29
$30 \leq n \leq 90$	0.51	0.56	0.44
$90 \leq n \leq 365$	0.92	0.63	1.11
$n \geq 365$	1.14	0.72	0.94

The cross-correlation coefficient ρ_{DCCA} between homicides and attempted homicides is positive at any time scale, then if we have an increase (decrease) in homicides, we expect an increase (decrease) in attempted homicides. This positive value says nothing about relationships of the series with lags/leads (in the cross-correlation sense), but it rather says that if the series moved together, they are likely to move together even during the following periods, for more information see Ref. [63].

4. Conclusions

This paper examined the time-series of daily homicides and attempted homicides in the city of Salvador (BR) from 2004 to 2011. Through the DFA, DCCA, and the cross-correlation coefficient ρ_{DCCA}, we identified in these time-series auto-correlation, cross-correlation (quantifying its level), and also seasonal components. DFA autocorrelation function oscillates between anti-persistent, persistent, and memoryless case, depending on the time scale in question. DCCA cross-correlation analysis showed that the time-series are cross-correlated, and using ρ_{DCCA}, we can see that cross-correlation is positive for all time scales. Thus, if the homicide increases, then the attempted homicide also increases, and vice versa. Using DFA, DCCA, and ρ_{DCCA}, we found several interesting properties at different time scales, mainly at the crosshatched column, Figs. 4 and 5, where there is a transition from anti-persistent to persistent behavior. ρ_{DCCA} has been shown to be appropriate for this analysis, because we can see the different time scales, and also remove the linear tendency.

We could also think how the air temperature influences the homicides or the attempted homicide, and measuring in this way the influence of climate on crime. The preliminary results, not presented in this paper, shows that the air temperature has no influence in the homicide or attempted homicide, in this case $\rho_{\mathrm{DCCA}} \approx 0$. Finally, this paper applied new techniques to study criminological data from police departments. This type of analysis proved to be robust in this treatment, because it could identify new seasonal components and quantify the level of cross-correlation between these data. In this sense, this paper explored new directions for crime study, and we can apply these methods to study other types of crimes, in order to aid the management of public security.

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